Solutions to Homework Problem Set 4

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3.1.11 a. $8\mathbb{P}$. **b.** $8\mathbb{P}$. **c.** $8\mathbb{P}$.

d. The surface is orientable with $v = 2, e = 5, f = 1$. So $\chi = -2$. Hence the surface is $2\mathbb{T}$.

e. The surface is nonorientable with $\chi = -2$. Hence the surface is $4\mathbb{P}$.

f,g. The surface is orientable with $\chi = -2$. So it is $2\mathbb{T}$.

h. The surface is nonorientable with $\chi = -5$. Hence it is $7\mathbb{P}$.

3.1.12 We have $e = 3$ and $f = 1$, and $1 \le v \le 6$. Hence $-1 \le \chi \le 4$. Since χ for surfaces is at most 2, we have $-1 \le \chi \le 2$. So the possible surfaces are $S^2, \mathbb{P}, \mathbb{T}, \mathbb{K}, 3\mathbb{P}$. These surfaces are indeed possible and words are, $abcc^{-1}b^{-1}a^{-1}, abcabc, aba^{-1}b^{-1}cc^{-1}, abab^{-1}cc^{-1}, aabbcc$, respectively.

3.1.14 The Euler characteristics for $M\#S^2$ is $\chi(M\#S^2) = \chi(M) + \chi(S^2) - 2 = \chi(M)$. Since M is orientable if and only if $M\#S^2$ is orientable, the surfaces M and $M\#S^2$ are homeomorphic.

3.1.15 $e = 82$ and $e = 80$.

3.1.16 $\chi(M\#\mathbb{P}) = \chi(M) + \chi(\mathbb{P}) - 2 = \chi(M) - 1$.

3.1.17 At the presence of $\mathbb{P}$ or $\mathbb{K}$, taking the connected sum with $\mathbb{T}$ is the same as the connected sum with $2\mathbb{P}$. So, $(k\mathbb{T})\#(m\mathbb{P}) = (2k + m)\mathbb{P}$, and $(n\mathbb{T})\#\mathbb{K} = (2n + 2)\mathbb{P}$. Thus, $m = 2n + 2 - 2k$. Similarly for (b).

3.1.18 The surface S we get after gluing x and y-edges have boundary circles. Let M be the surface obtained by capping these boundary circles by discs. Then M is a compact path connected surface without boundary, and it must be of the form $n\mathbb{T}$ or $m\mathbb{P}$. To identify S, we first identify M. We note that
S is orientable if and only if M is orientable,
$\chi(M) = \chi(S) + k$, where k is the number of discs attached to S to close the boundaries. To compute $\chi(S)$, we consider a cell decomposition of the surface S in a simple way and use $\chi(S) = v - e + f$.

We consider three cases. (i) Both x and y edges are glued in the same direction so that the resulting surface S is orientable. (ii) Glue one of x or y edges with a half twist so that the resulting surface is nonorientable (S has a Möbius band inside). (iii) Glue both x and y-edges with half twists so that the surface S is nonorientable.

In all three cases above, $\chi(S) = -1$, and in the first two cases there is one boundary for S, and in the third case there are two boundary circles. Hence $\chi(M)$ is 0 for (i) and (ii), and 1 for (iii). Hence M is $\mathbb{T}, \mathbb{K}, \mathbb{P}$ respectively. The original surfaces are obtained by removing an open disc from M for (i) and (ii), and two open discs for (iii).

3.4.9 Klein bottle is basically "half" of torus. Try to cut the regular complex for torus into half so that edges match.

3.4.11, 12 We have a cell decomposition of M of type $(3, k)$, so we have $2e = 3f$ and $kv = 2e$. We have $\chi(M) = v - e + f = 2e(6 - k)/6k$. If $k \leq 6$, then $\chi(M) \geq 0$. Hence $M = S^2, \mathbb{P}, \mathbb{T}, \mathbb{K}$.

Hence $2\mathbb{T}$ cannot have a triangulation of type $(3, k)$ with $k < 7$.

3.4.13 We can have regular complexes of type $(3, 6), (4, 4), (6, 3)$. Type $(6, 3)$ is obtained by taking dual of $(3, 6)$.