Quiz 1, Math 124, Fall 2009

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Due: Thursday October 29, in class

Instruction: *I suggest that you type your solution using LaTeX. Clarity and precision of your writing is part of your scores. Your solution is your first draft. Reread it and correct your English. Treat this take-home quiz as your writing assignment.*

[**1**] The M be a surface whose plane model is given by a word

$$abcdf^{-1}d^{-1}fg^{-1}cgee^{-1}b^{-1}a^{-1}.$$

(1) Is M orientable?
(2) Use circulation rules to transform this word into a standard form, and identify this surface as $n\mathbb{T}$, or $m\mathbb{P}$.

[**2**] Give precise definitions of the following concepts. Statements equivalent to definitions are acceptable.

(1) A continuous map from a topological space X to a topological space Y.
(2) A compact topological space.
(3) A Hausdorff topological space.
(4) A connected topological space
(5) A limit point of a subset A of a topological space X.

[**3**] Answer the following questions by yes or no.

(1) A torus with a finitely many distinct points removed is compact.
(2) Closed subsets in a Hausdorff space are compact.
(3) Compact subsets of a toplogical space are closed.
(4) Let $Y \subset \mathbb{R}^2$ be the graph of $y = x^2$ for $x \geq 0$. That is $Y = \{(x, x^2) \mid x \geq 0\}$. Then Y is a compact subset of $\mathbb{R}^2$.
(5) The set of all rational numbers contained in the closed interval $[0, 1]$ is compact.
(6) Topologist's closed sine curve is a subset of $\mathbb{R}^2$ given by $\{(x, \sin\frac{1}{x}) \mid 0 < x \leq 1\} \cup \{(0, y) \mid -1 \leq y \leq 1\}$. This subset is a compact subset of $\mathbb{R}^2$.
(7) Let $f : X \to Y$ be a continuous onto map from a compact space X to a space Y. Then Y is necessarily compact.
(8) Hawaiian earring is a subset of $\mathbb{R}^2$ given by

$$\bigcup_{n\geq 1}\{(x, y) \mid (x - \frac{1}{n})^2 + y^2 = (\frac{1}{n})^2\}.$$

This is a union of circle converging to the origin. The hawaiian earring is a compact topological space.

(9) A compact subset of $\mathbb{R}^n$ is bounded.

(10) Let h be a homeomorphism from the boundary of a closed 2-dimensional disc D^2 to the boundary of a Möbius strip M. Identify the boundary points of D^2 and M by h. The resulting space is a projective plane P^2.

(11) Let Y be a subspace of a topological space X. Let A be a subset of Y. If A is open in Y and Y is open in X, the A is necessarily an open subset of X.

(12) The Klein bottle and the projective plane are compact topological spaces.

(13) The subset $A = \{(x, y) \in \mathbb{R}^2 \mid y = 0\}$ of $\mathbb{R}^2$ is an open subset of $\mathbb{R}^2$.

(14) Let X be a Hausdorff space and let A be a compact subset and let $x \notin A$. Then there exists a disjoint open neighborhoods of A and x.

(15) Let X be a Hausdorff space and let A, B be disjoint compact subsets. Then there exist disjoint open subsets U and V of X such that $A \subset U$ and $B \subset V$.

(16) Suppose a continuous map $f : X \to Y$ is 1:1 and onto. If f maps closed subsets to closed subsets, then f is a homeomorphism.

[**4**] Let X be a topological space, and let A and B be subsets of X. Prove the following statements for closures.

(1) Show that if $A \subset B$, then $\overline{A} \subset \overline{B}$ in X

(2) Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$ in X.

[**5**] Prove that a continuous map $f : X \to Y$ from a compact space X to a Hausdorff space Y is closed, that is, for every closed subset V of X, its image $f(V)$ is a closed subset in Y.

[**6**] Let X be the set of real numbers $\mathbb{R}$ with finite complement toplogy.

(1) Describe open sets of finite complement topology on the set $\mathbb{R}$.

(2) Is X a Hausdorff topological space? Explain.

(3) Show that X is a compact topological space.

(4) Show that every subset of X is a compact subset.