MATH 19B, Integral Calculus: On Partial Fractions:

1. Every rational function

$$f(x) = \frac{P(x)}{Q(x)}$$

where the degree of $P(x)$ is less than the degree of $Q(x)$ can be rewritten as a sum of partial fractions. To do this, you must first factor $Q(x)$ into linear and possibly irreducible quadratic factors, each of which may or may not be repeated.

Then the contribution of each such factor of $Q(x)$ is as follows:

(a) A linear factor

$$(ax + b)$$

of $Q(x)$ that is not repeated contributes a partial fraction

$$\frac{A}{ax + b}$$

(b) A linear factor

$$(ax + b)^r$$

of $Q(x)$ that is repeated r times contributes r partial fractions

$$\frac{A_1}{(ax+b)} + \frac{A_2}{(ax+b)^2} + \frac{A_3}{(ax+b)^3} + \cdots + \frac{A_r}{(ax+b)^r}$$

(c) An irreducible quadratic factor

$$(ax^2 + bx + c)$$

of $Q(x)$ that is not repeated contributes a partial fraction

$$\frac{Ax + B}{ax^2 + bx + c}$$

(d) An irreducible quadratic factor

$$(ax^2 + bx + c)^r$$

of $Q(x)$ that is repeated r times contributes r partial fractions

$$\frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \frac{A_3x + B_3}{(ax^2 + bx + c)^3} + \cdots + \frac{A_rx + B_r}{(ax^2 + bx + c)^r}$$